

Asymptotically Static Universe

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Abstract

We consider a cosmology in which the final stage of the Universe is neither accelerating nor decelerating, but approaches an asymptotic state where the scale factor becomes a constant value. In order to achieve this, we first bring in a scale factor with the desired property and then determine the details of the energy contents as a result of the cosmological evolution equations. We show that such a scenario can be realized if we introduce a generalized quintom model which consists of a scalar field and a phantom with a *negative* cosmological constant term. The standard cold dark matter with $w_m = 0$ is also introduced. This is possible basically due to the balance between the matter and the *negative* cosmological constant which tend to attract and scalar field and phantom which repel in the asymptotic region. The stability analysis shows that this asymptotic solution is classically stable.

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1 Introduction

Recent observational data [1, 2] suggests that dark energy is believed to be responsible for the observed acceleration of the Universe [3]. Many theoretical proposals on the nature of dark energy have been put forward. Cosmological constant [4, 5, 6, 7, 8] is widely accepted as the most promising candidate for dark energy, but there is the problem of its extreme fine-tuning. There are many alternative approaches which do not contradict with observational data [9]. One possible approach is based on the fact that the matter with the equation of state parameter $w < -1$ is not ruled out by observational data and moreover necessary to describe the current acceleration of the universe. Phantom [10] is an alternative form of dark energy with this property and various aspects have been discussed extensively [11, 12, 13, 14].

In spite of many theoretical attempts, at present the nature of the dark energy is still enigmatic. An alternative approach to dark energy could be to try the "reverse" way; Instead of guessing the energy density and then trying to determine the evolution of the universe, one first assumes some specific scale factor $a(t)$, and then fix the energy density from the equations of motion [15, 16, 17, 18]. This type of approach was proposed before in which the focus has been primarily on the description of the present universe starting from the past. In this work, we will be interested in the future of the universe. So far, many results of the existing literatures on the future of the Universe is based on the consequences of acceleration [19, 20, 21].

However, the ignorance of the exact content of the dark energy leaves other choices of the scale factor as viable. In this paper, we will introduce a particular form of the scale factor which is consistent with the present accelerating universe. It represents the universe which does not expand forever, but approaches asymptotically a constant value¹. The motivation for considering such Universe is that there is a priori no reason why the acceleration of the universe should be the final state. The Λ -CDM suggests the acceleration as a final stage, but other contents of the dark energy could give different fate of the Universe. For example, in the phantom model the energy density may increase with time and the Universe ends up in big-rip singularity [11, 12, 26] in which the Hubble parameter and the curvature diverges in a finite time. Another speculation is that dark energy might dissipate with time and Universe contracts in a big-crunch [27, 28] as in

¹The difference with the Einstein static universe is that in this case, the universe stays static or starts from a static state [22], whereas in our case the static state is approached asymptotically in the future. It can be shown that the Einstein static universe is stable against inhomogeneous perturbations [23, 24, 25], even though it is unstable when the homogeneous perturbations are performed. We only consider homogeneous perturbations in this work.

the cyclic model [29, 30, 31]. Here, we investigate Universe neither accelerating nor decelerating, but becoming asymptotically static with its size approaching a constant value, and ask what type of energy contents will yield such Universe.

In order to implement the idea, we consider a generalized quintom model [32, 33, 34] which consists of a scalar field and phantom, and also the cold dark matter which satisfies the continuity equation. The potentials of the scalar fields and other quantities defining the model are not fixed at this stage. With this, we bring in a scale factor which has the property that it is accelerating at present, but begins to decelerate after a fixed time, and eventually settles down to an asymptotic value. Then, the details of the theory are determined through the evolution equations. We show that such a scenario is possible due to the balance between the attractive matter and the negative cosmological constant, and the repulsive scalar field and phantom in the asymptotic region. The stability analysis shows that this asymptotic solution is classically stable.

2 Phantom model

Let us first consider phantom cosmology where gravity is coupled to a scalar field with a scalar coupling function and a potential. This model was considered before [16] where the possibility of phantom-non-phantom transition occurs in early time as well as at late time. In this paper, we go one step further and discuss the future of the Universe with such a scalar coupling function.

The action is given by

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{2} \tilde{\omega}(\varphi) \nabla_\mu \varphi \nabla^\mu \varphi - V(\varphi) \right\} + S_m, \quad (2.1)$$

where $\kappa^2 \equiv 8\pi G_N$ is the gravitational coupling, $\tilde{\omega}(\varphi)$ can change continuously, and $V(\varphi)$ is the potential. S_m is the matter action which we assume to be a cold dark matter with $w_m = 0$, and satisfies the continuity equation. The scalar field φ is a function of time alone. The evolution equations are given by

$$\begin{aligned} \frac{3}{\kappa^2} H^2 &= \rho_\varphi + \rho_m, \\ -\frac{2}{\kappa^2} \dot{H} &= p_\varphi + \rho_\varphi + \rho_m, \end{aligned} \quad (2.2)$$

where $H = \dot{a}/a$ is the Hubble parameter, the energy density ρ_φ and the pressure p_φ are

$$\begin{aligned} \rho_\varphi &= \frac{1}{2} \tilde{\omega}(\varphi) \dot{\varphi}^2 + V, \\ p_\varphi &= \frac{1}{2} \tilde{\omega}(\varphi) \dot{\varphi}^2 - V, \end{aligned} \quad (2.3)$$

and the scalar field satisfies

$$\tilde{\omega}(\varphi)\ddot{\varphi} + \frac{1}{2}\tilde{\omega}'(\varphi)\dot{\varphi}^2 + 3H\tilde{\omega}(\varphi)\dot{\varphi} + \frac{\partial V(\varphi)}{\partial \varphi} = 0. \quad (2.4)$$

There could be many expressions of scale factor which has the desired property of transition from acceleration to deceleration. In this work, we choose a special ansatz of the scale factor as follows;

$$a(t) = a_\infty(1 - Ae^{-B(t/t_0)^n}), \quad (2.5)$$

where A, B are constants, t_0 is the current age of the Universe, a_∞ is the asymptotic value, and n is arbitrary. This ansatz shows the behavior that the Universe accelerates until some time (given below) and then deceleration takes place until it reaches an asymptotic value. Therefore, it does not fit the whole history of the Universe, and we assume it is an effective scale factor which describes the Universe only after the late-time acceleration has begun, $t_{tr} \simeq 0.5t_0$. The transition from acceleration to deceleration will happen at the time t_* satisfying the following conditions:

$$\ddot{a}(t)\Big|_{t=t_*} = 0, \quad t_*/t_0 = \sqrt[n]{\frac{n-1}{nB}}. \quad (2.6)$$

The condition that the deceleration must take place at later time restricts $n > 1/(1-B)$.

With this scale factor, the Hubble parameter is given by

$$H(t) = \frac{1}{t_0} \frac{ABn(t/t_0)^{n-1}e^{-B(t/t_0)^n}}{1 - Ae^{-B(t/t_0)^n}},$$

and the matter density is given as

$$\rho_m(t) = \frac{\rho_0(1 - Ae^{-B(t/t_0)^n})^3}{(1 - Ae^{-B(t/t_0)^n})^3}.$$

With the ansatz as in [16]

$$\varphi = t, \quad (2.7)$$

the Einstein equations (2.2) can be rewritten as

$$\begin{aligned} \tilde{\omega}(t) &= -\frac{2}{\kappa^2}\dot{H}(t) - \rho_m(t), \\ V(t) &= \frac{1}{\kappa^2}\left(3H^2(t) + \dot{H}(t)\right) - \frac{1}{2}\rho_m(t). \end{aligned}$$

In order to determine the constant A and B , we demand

$$\begin{aligned} a(t_0) &= a_\infty(1 - Ae^{-B}) \equiv a_0, \\ H(t_0) &= \frac{1}{t_0} \frac{ABne^{-B}}{1 - Ae^{-B}} \simeq \frac{1}{t_0}. \end{aligned}$$

Then, we get the following two relations

$$A = (1 - \frac{1}{m})e^B, \quad B = \frac{1}{n(m-1)}$$

where $m \equiv \frac{a_\infty}{a_0}$.

The free parameters are given by m and n ; m determined the final size of the universe, and n is related with the transition time from acceleration to deceleration. They can be restricted by the observational data. The effective equation of state parameter w_φ is given by

$$w_\varphi = \frac{p_\varphi}{\rho_\varphi} = \frac{\frac{\tilde{\omega}(t)}{2} - V(t)}{\frac{\tilde{\omega}(t)}{2} + V(t)}, \quad (2.8)$$

and at the current time we confront the equation of state parameter w_φ with experimental uncertainty ($-1.14 \leq w_\varphi \leq -0.88$) [2] which constrains the parameter n and m . The numerical analysis is given in Fig. 1 which shows that there exists allowed region within the experimental uncertainty. Among the possible values, we choose a point and present numerical result in Fig. 2. The result shows that at current time, the potential is dominant so that the Universe is accelerating. Also the scalar function $\tilde{\omega}$ is slightly negative and it is in the phantom phase. As it evolves, the deceleration begins to take over as the scalar function $\tilde{\omega}(\varphi)$ (kinetic energy) reaches its near peak value. Then, both the potential and the scalar function decrease rapidly and becomes negative. However, the matter density stays always positive, decreasing as $1/a^3$, and asymptotically it cancels exactly with the scalar contributions of the potential and the kinetic energy. This exact cancelation renders the Universe asymptotically static.

In Fig. 2, we plot various quantities for values of $(m, n) = (3.5, 2)$.

3 STABILITY OF THE PHANTOM FIELD

Let us check the stability of the ansatz (2.5) and (2.7). In fact, this model(with one scalar field) could be unstable due to several reasons. The first one is that the effective equation of state parameter diverges, which occur in case of vanishing of denominator in

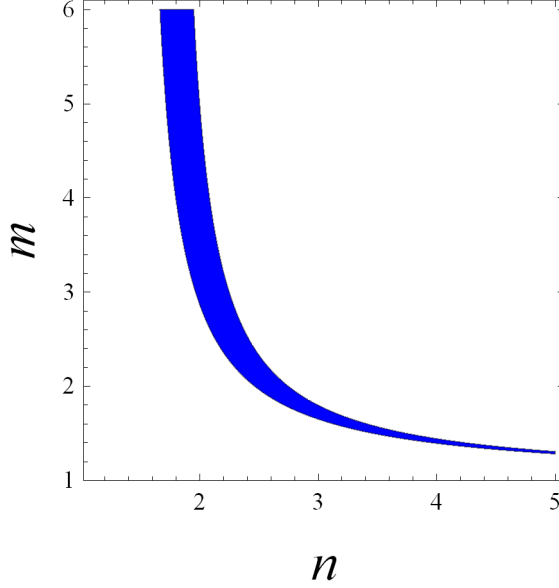


Figure 1: The experimental data constraints to the (n, m) parameter space. The blue shaded region are allowed by experimental uncertainty $-1.14 \leq w_\varphi \leq -0.88$. (We set the scale factor $a_0 = 1$, current time $t_0 = 1$, and $\kappa = 1$.)

(2.8). The other one is the instability of the solution (2.5) and (2.7) due to singularity which occurs when crossing the point of the coefficient of kinetic term $\tilde{\omega}(\varphi)$ becoming zero as discussed in [16]. In order to avoid these problems, we may consider two scalar fields model like

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} (R - 2\Lambda) - \frac{1}{2} \omega(\phi) \nabla_\mu \phi \nabla^\mu \phi - \frac{1}{2} \eta(\chi) \nabla_\mu \chi \nabla^\mu \chi - V(\phi, \chi) \right\} + S_m \quad (3.9)$$

where $V(\phi, \chi)$ are potential energy for which we assume that there is no direct coupling between ϕ field and χ field, $V(\phi, \chi) = V_\phi(\phi) + V_\chi(\chi)$. Λ is a cosmological constant. For our purposes, we take the value of Λ to be negative.

By varying the action, one can obtain the Einstein equations and the continuity equation for CDM as follows;

$$\begin{aligned} \frac{3}{\kappa^2} H^2 &= \rho_\phi + \rho_\chi + \rho_m + \frac{\Lambda}{\kappa^2}, \\ -\frac{2}{\kappa^2} \dot{H} &= \rho_\phi + \rho_\chi + p_\phi + p_\chi + \rho_m, \end{aligned} \quad (3.10)$$

$$0 = \dot{\rho}_m + 3H\rho_m, \quad (3.11)$$

where energy density ρ_ϕ , ρ_χ and pressure p_ϕ , p_χ are given by

$$\rho_\phi = \frac{1}{2} \omega(\phi) \dot{\phi}^2 + V_\phi, \quad \rho_\chi = \frac{1}{2} \eta(\chi) \dot{\chi}^2 + V_\chi,$$

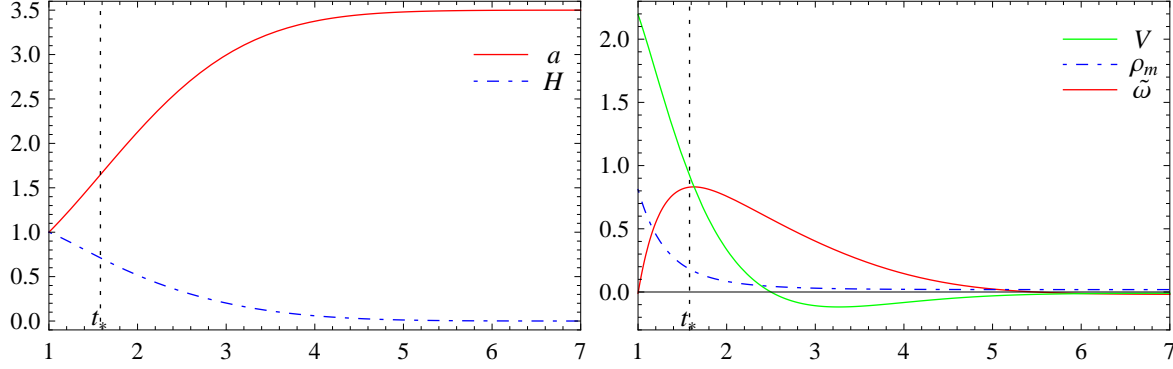


Figure 2: The left graphs are time evolution of the $a(t)$ (solid curve) and $H(t)$ (dash – dotted curve). The right graphs are time evolution of the $V(t)$ (green solid curve), $\rho_m(t)$ (dash – dotted curve), and $\tilde{\omega}$ (red solid curve). (The parameter values are used for $n = 2$ and $m = 3.5$. We set the scale factor $a_0 = 1$, current time $t_0 = 1$, and $\kappa = 1$.) In these figures, the Universe accelerates until the time $t_* \equiv 1.58$.

$$p_\phi = \frac{1}{2}\omega(\phi)\dot{\phi}^2 - V_\phi, \quad p_\chi = \frac{1}{2}\eta(\chi)\dot{\chi}^2 - V_\chi, \quad (3.12)$$

and equations of scalar fields ϕ and χ as

$$\begin{aligned} 0 &= \omega(\phi)\ddot{\phi} + \frac{1}{2}\omega'(\phi)\dot{\phi}^2 + 3H\omega(\phi)\dot{\phi} + \frac{\partial V_\phi(\phi)}{\partial \phi}, \\ 0 &= \eta(\chi)\ddot{\chi} + \frac{1}{2}\eta'(\chi)\dot{\chi}^2 + 3H\eta(\chi)\dot{\chi} + \frac{\partial V_\chi(\chi)}{\partial \chi}. \end{aligned} \quad (3.13)$$

In the two scalar fields model, we can choose the corresponding coefficients ω and η to be non-vanishing everywhere. Then the instability due to divergence would not occur. For example, we may choose that ω should be always positive and η be always negative [16],

$$\begin{aligned} \omega(\phi) &= \sqrt{\alpha^2 + \tilde{\omega}(\phi)^2}, \\ \eta(\chi) &= \tilde{\omega}(\chi) - \sqrt{\alpha^2 + \tilde{\omega}(\chi)^2}, \end{aligned} \quad (3.14)$$

where α is a non-zero constant. Since ω and η are positively- and negatively-defined, respectively, ω gives ordinary scalar phase and η phantom phase. The numerical evaluation is shown in Fig. 3. The corresponding potential energy $V_\phi(\phi)$ and $V_\chi(\chi)$ are obtained by solving the following differential equations that are just the scalar field equations (3.13) with solution $\phi = t$ and $\chi = t$

$$\dot{V}_\phi = -\frac{1}{2}\dot{\omega} - 3H\omega,$$

$$\dot{V}_\chi = -\frac{1}{2}\dot{\eta} - 3H\eta, \quad (3.15)$$

where the solutions V_ϕ and V_χ are generally dependent upon integration constant which can be defined in accordance with the potential energy of the one scalar fields model.

The corresponding effective equation of state parameter w_ϕ and w_χ are given by

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{\omega(t)}{2} - V_\phi(t)}{\frac{\omega(t)}{2} + V_\phi(t)}, \quad w_\chi = \frac{p_\chi}{\rho_\chi} = \frac{\frac{\eta(t)}{2} - V_\chi(t)}{\frac{\eta(t)}{2} + V_\chi(t)}. \quad (3.16)$$

As was mentioned before, the effective equation of state parameter of the one scalar field model becomes singular at some point. In order to obtain the well defined effective equation of state parameter of the two scalar field model, we can choose the integration constant such that denominator in (3.16) should be always non-vanishing. This can be done and some numerical result is shown in Fig. 4

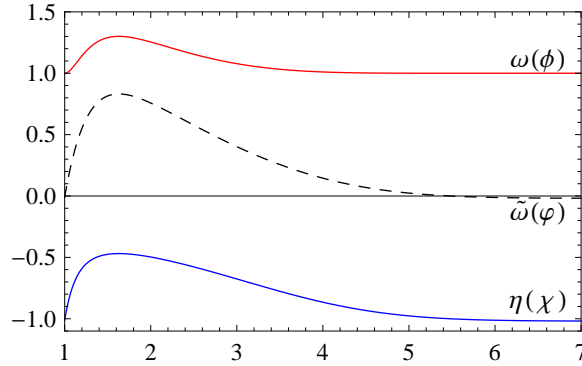


Figure 3: The graphs are evolution of the coefficient of the kinetic term $\omega(\phi)$ and $\eta(\chi)$. The dashed curve is coefficient of the kinetic term $\tilde{\omega}(\varphi)$ of the one scalar field model. (We set the parameter value $\alpha = 1$).

To gain some insight into the property of the solution, we introducing the following variables [16]:

$$X_\phi \equiv \dot{\phi}, \quad X_\chi \equiv \dot{\chi}, \quad (3.17)$$

and we consider the perturbations of variables X_ϕ, X_χ, H , and ρ_m near the background solution. One can obtain the equations for the linear perturbations up to the first order as follows:

$$\frac{d}{dt}\delta X_\phi = \left(-\frac{\omega'}{\omega} - 3H\right)\delta X_\phi - 3\delta H,$$

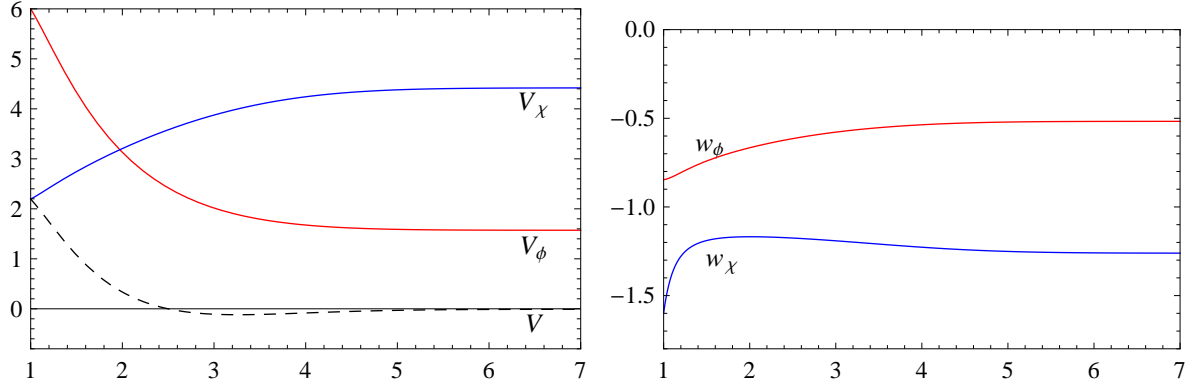


Figure 4: The left graphs are evolution of the potential energy of V_ϕ (red curve) and V_χ (blue curve). The dashed curve is evolution of the potential energy of the one scalar field model. The right graphs are evolution of the effective equation of state of field ϕ (red curve) and field χ (blue curve). (We choosing the initial condition $V_\phi(1) = 6$ and $V_\chi(1) = V(1) - V_\phi(1) - \Lambda/\kappa^2$ with $\Lambda = -6$ and $\kappa^2 = 1$.)

$$\begin{aligned}
\frac{d}{dt}\delta X_\chi &= \left(-\frac{\eta'}{\eta} - 3H\right)\delta X_\chi - 3\delta H, \\
\frac{d}{dt}\delta H &= -\omega\delta X_\phi - \eta\delta X_\chi - \frac{1}{2}\delta\rho_m, \\
\frac{d}{dt}\delta\rho_m &= -3\rho_m\delta H - 3H\delta\rho_m.
\end{aligned} \tag{3.18}$$

Before we carry out the numerical analysis, let us analyze the late time property analytically. When time t becomes very large, the contribution of H, ω' , and η' will become negligible. Therefore Eq.(3.18) reduces to

$$\frac{d}{dt} \begin{pmatrix} \delta X_\phi \\ \delta X_\chi \\ \delta H \\ \delta\rho_m \end{pmatrix} = M \begin{pmatrix} \delta X_\phi \\ \delta X_\chi \\ \delta H \\ \delta\rho_m \end{pmatrix}, \quad M \equiv \begin{pmatrix} 0 & 0 & -3 & 0 \\ 0 & 0 & -3 & 0 \\ -\omega_\infty & -\eta_\infty & 0 & -\frac{1}{2} \\ 0 & 0 & -\rho_{m,\infty} & 0 \end{pmatrix}, \tag{3.19}$$

where $\omega_\infty, \eta_\infty$ and $\rho_{m,\infty}$ are values at late time. The eigenvalues of matrix M are given by

$$0, 0, \pm\sqrt{3\left(\omega_\infty + \eta_\infty + \frac{1}{2}\rho_{m,\infty}\right)}. \tag{3.20}$$

Since $\omega_\infty + \eta_\infty + \frac{1}{2}\rho_{m,\infty} \simeq -\frac{\rho_0}{2m^3}$ is negative (for the positive value m), there are no positive eigenvalues. Thus, the solution is stable in the late time. We mention that this

result can also be obtained by solving the differential equations Eq.(3.19), and in the late time region, the solutions are

$$\begin{aligned}
\delta X_\phi(t) &= A_\phi + B \cos(\lambda t) + C \sin(\lambda t), \\
\delta X_\chi(t) &= A_\chi + B \cos(\lambda t) + C \sin(\lambda t), \\
\delta H(t) &= \frac{\lambda}{3} \left(-C \cos(\lambda t) + B \sin(\lambda t) \right), \\
\delta \rho_m(t) &= A_{\rho_m} + \rho_\infty (B \cos(\lambda t) + C \sin(\lambda t)),
\end{aligned} \tag{3.21}$$

where λ , A_ϕ , A_χ , A_{ρ_m} , B , and C are given by

$$\begin{aligned}
\lambda &= \sqrt{-3 \left(\omega_\infty + \eta_\infty + \frac{1}{2} \rho_{m,\infty} \right)}, \\
A_\phi &= \frac{3}{2\lambda^2} \left(I_4 + 2(I_2 - I_1)\eta_\infty - I_1 \rho_{m,\infty} \right), \\
A_\chi &= \frac{3}{2\lambda^2} \left(I_4 - I_2 \rho_\infty + 2(I_1 - I_2)\omega_\infty \right), \\
A_{\rho_m} &= \frac{3}{2\lambda^2} \left(2(I_2 \eta_\infty + I_1 \omega_\infty) \rho_{m,\infty} - 2I_4(\eta_\infty + \omega_\infty) \right), \\
B &= -\frac{3}{2\lambda^2} \left(I_4 + 2I_2 \eta_\infty + 2I_1 \omega_\infty \right), \\
C &= -\frac{3I_3}{\lambda},
\end{aligned} \tag{3.22}$$

with I_1 , I_2 , I_3 , and I_4 being integration constants. We find that the solutions has an oscillating behavior with finite amplitude at late times. A direct numerical analysis of the Eq. (3.18) also confirms the stability. The results are shown in Fig. 5. It shows that the perturbation settles down to the one given in Eq. (3.21) very rapidly. The subsequent variation oscillates around the solution.

4 Conclusion

In this paper, we considered a generalized quintom model which consists of a scalar field and phantom, and also the cold dark matter which satisfies the continuity equation. The potential of the scalar fields and other quantities defining the model are determined by the condition that they are consistent with the scale factor which has the property that it is accelerating at present, but begins to decelerate after a fixed time, and eventually settles down to an asymptotic value. The details of the theory are determined through the evolution equations. We showed that the asymptotic scenario is possible due to the balance between the matter and the negative cosmological constant which tend to attract

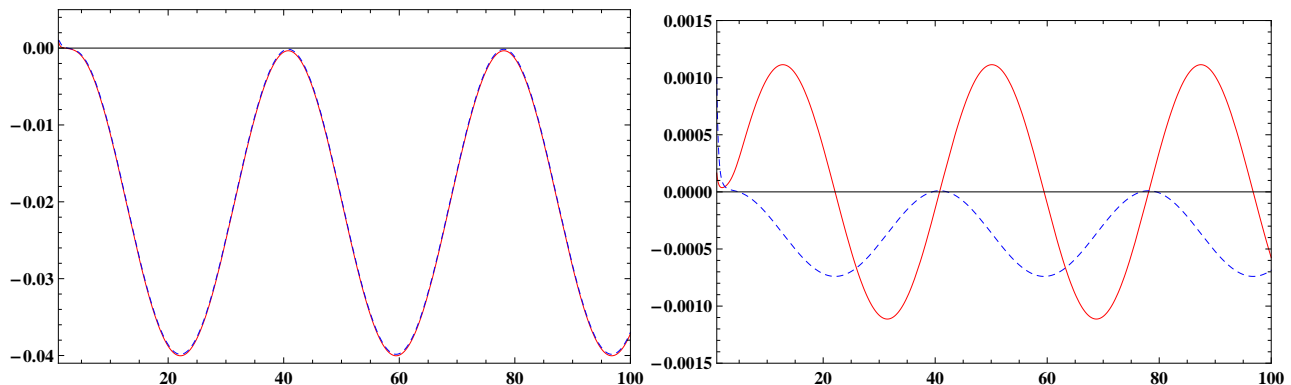


Figure 5: The left graphs are time evolution of the δX_ϕ (solid curve) and δX_χ (dashed curve). The right graphs are time evolution of the δH (solid curve) and $\delta \rho_m$ (dashed curve). (The parameter values used are $n = 2, m = 3.5$ and $\alpha = 1$ and computation done by choosing the initial conditions $\delta X_\phi = 0.001, \delta X_\chi = 0.001, \delta H = 0.000165$, and $\delta \rho_m = 0.001$.)

and scalar field and phantom which repel in the asymptotic region. The stability analysis shows that this asymptotic solution is classically stable.

The scale factor showing the behavior of approaching an asymptotic state is by no means unique. We could try other ansatz which exhibits similar property with the one investigated in this paper. Obviously, for different choices of the scale factor, the energy contents will be given differently. Nevertheless, the over-all feature of necessity of the phantom will remain, because in order to achieve the asymptotic final stage, the attractive force of the matter must balance the repulsive phantom. It remains to be seen whether the exact nature of the dark energy could support such an asymptotically static Universe.

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